

Integrated Water Resources Management in the Lower Jordan Rift Valley

Sustainable Management of Available Water Resources with Innovative Technologies



Work package 6, Deliverables D601

DSS Development

J. Bensabat, M. Kitron-Belinkov, T. Milgrom, S. Dror

Feb, 2012

Corresponding author:

Jacob Bensabat

EWRE Environmental & Water Resources Engineering Ltd.

PO Box 6770 Haifa 31067, Israel. jbensabat@ewre.com , 972-4-838-3919

Funded by the German Federal Ministry of Education and Research (BMBF):

FKZ 02WM0801

Project Coordination:**Main Coordinator:**

Prof. Dr. Heinz Hötzl
Department of Applied Geology
Karlsruhe University
Adenauerring 20b
76131 Karlsruhe
Germany
Phone: +49 (0) 721 608 3096
Fax: +49 (0) 721 606 279

Assistant Coordinators

Prof. Dr. Martin Sauter
Department of Applied
Geology
Göttingen University
Goldschmidtstrasse 3
37077 Göttingen
Germany
Phone: +49 (0)551 39 79 11
Fax: +49 (0)551 39 93 79

Dr. Stefan Geyer & Dr. Roland Mueller
Helmholtz Centre for Environmental
Research (UFZ)
Permoserstr. 15
04318 Leipzig
Germany
Phone: +49 (0)341 235 30 00
Fax: +49 (0)341 235 2885

<http://www.iwrm-smart.org/>

TABLE OF CONTENTS

Table of Contents	3
List of Figures	3
List of Tables	3
1 Introduction	4
2 The Analytical Hierarchy Process (AHP) Concept	6
2.1. AHP Algorithm	7
3 AHP User's Guide	9
4 Electre Tri Algorithm	14
4.1 Overall view	14
4.2 The Algorithm in detail	15
4.2.1 Definitions of parameters	15
5 7. Electre Tri Assistant Algorithm	20
6 7.1 Overall view	20
7 7.2 The Algorithm in detail	21
8 8. ELECTRE TRI Assistant User's Guide	22

LIST OF FIGURES

Figure 1: Illustration of the Analytic Hierarchy process	6
Figure 3: A simplified geometric model of the Electre Tri sorting problem.	14
Figure 4: Graphic representation of a criterion and associated parameters.	15
Figure 5: Illustration of an optimization problem	20

LIST OF TABLES

Table 1: A nine point scale for pair-wise comparison

1 Introduction

Decisions are typically characterized by a large set of alternatives and multiple, conflicting and incommensurate evaluation criteria. Multi Criteria Decision Making (MCDM) is a discipline aimed at supporting decision makers who are faced with making numerous and conflicting evaluations (criteria). This methodology aims at highlighting these conflicts and deriving a way to come to a compromise in a transparent process, mainly lead by the decision maker. MCDM is the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process.

The diversity of MCDM models makes it difficult to gain a clear understanding of which methodologies are appropriate to a particular context. Therefore, the choice of which model is most appropriate depends on the problem at hand and may be to some extent dependent on which model the decision maker is most comfortable with.

In order to make decisions in the complex environment of multiple criteria, we need to know the problem, the need and purpose of the decision (the goal), the criteria with which we screen the decisions, to define a set of sub-criteria, consult the stakeholders and other affected groups and finally to define a set of alternatives or scenarios that can meet the stated goal.

We then try to determine the best alternative, or in the case of resource allocation, we need priorities for the alternatives to allocate their appropriate share of the resources. The goals of a certain project or action have to be defined by decision variables and need to be maximized subject to constraints.

Problem involving multiple criteria can be divided into three types: **choice** problems – where a small (possibly one) subset of alternatives should be chosen out of a large number, **sorting** problems – where each alternative should be assigned to ~~one~~ a single category out of a predefined set, and **ranking** problems – where the alternatives are compared and ordered.

MCDM problems involve a number of criteria that may not be of equal importance to the decision maker. The decisions involve many intangibles that need to be traded off. Through tradeoffs it clarifies the advantages and disadvantages of policy options under circumstances of risks and uncertainty. Therefore, each criterion is assigned a weight that represents the preference of the decision maker with regard to the objectives. This represents the main core of the Analytical Hierarchy Process (AHP), which was developed by Thomas L. Saaty in the 1970's and has been extensively studied and refined since then.

The AHP is a mathematical theory of value, reason, and judgment, based on ratio scales for the analysis of multiple-criteria decision-making problems (Saaty, 2008). The AHP method is designated to assist the decision maker to **rank** predefined alternatives according to a set of predefined criteria, following a pair-wise comparison process (Saaty 2001; Saaty 2005; Saaty 2006; Saaty and Hu 1998; Barzilai and Golany 1994). It is important to note that some combinations of alternatives are contrary to the others and these tools are aiming in defining and selecting the proper alternative (combination).

Electre Tri is another MCDM tool. Like AHP, it involves selection of alternatives according to criteria to which weights are assigned. Unlike AHP, the goal of the process is not a single alternative to be chosen as the best one. Rather, it is **sorting** a number of alternatives to predefined categories. The categories are ordered from worst to best, of course, but the result could be that there are a number of alternatives such that all belong to the best category.

2 The Analytical Hierarchy Process (AHP) Concept

The AHP is a systematic procedure to construct and represent the elements of a problem in a hierarchy formant. The basic rationale of AHP is organized by the breakdown of the problem into smaller constituent parts at different levels. An AHP model typically consists of an overall **goal**, a set of **criteria** to specify the overall goal decomposed to sub-criteria, and finally, at the lowest level of the hierarchy, the decision **alternatives** to be evaluated. In AHP, each element in the hierarchy is considered to be independent of all the others. The hierarchy can be visualized as a diagram like the one below (Figure 1).

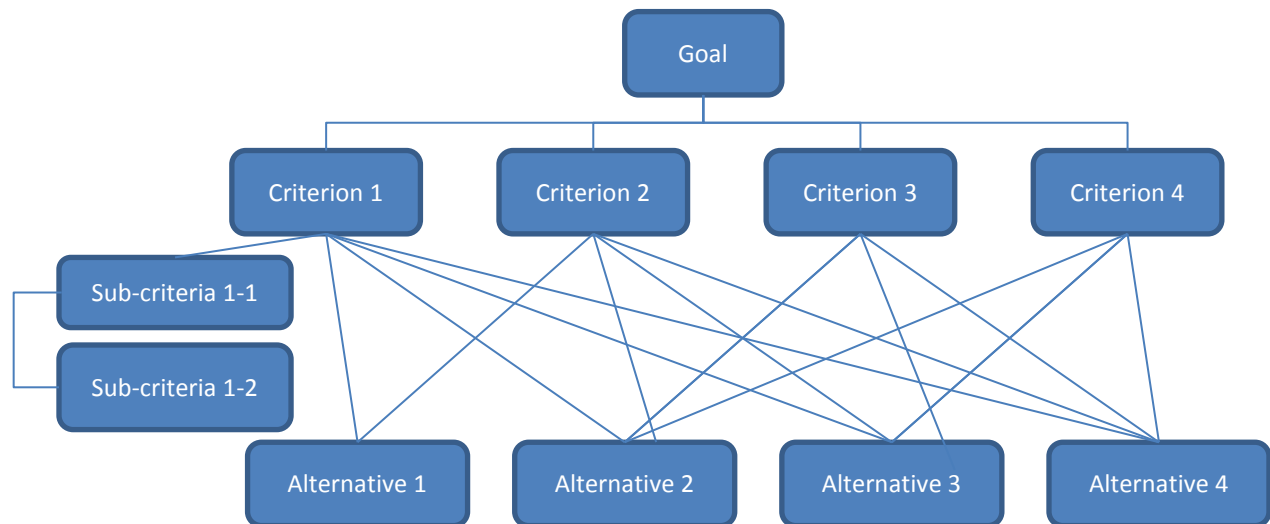


Figure 1: Illustration of the Analytic Hierarchy process

Beyond the decomposition principle, the AHP is based on pair-wise comparisons of elements in a decision hierarchy with respect to the parent element at the next higher hierarchical level (i.e., among criteria and lower level elements). The pair-wise comparison is aimed providing a cardinal scale to evaluate objects according to some subjective preference criteria. Pair-wise comparisons are made on a scale of relative importance (see Table 1) where the decision maker has the option to express the preferences between two elements on a ratio scale from equally important (i.e., equivalent to a numeric value of one) to absolute preference (i.e., equivalent to a numeric value of nine) of one element over another.

Ratings of decision makers are arranged as numerical numbers in a comparison matrix. Based on this, relative weights for all elements of the hierarchy are calculated with the eigenvector method (**EVM**), indicating the priority level for each element in the hierarchy (Saaty, 2001). Accordingly, priorities for the alternatives are gained by judgments with respect to each above-level element of the hierarchy. Their performances are weighted with the relative weights of criteria and sub-criteria, and added to an overall priority for each alternative (i.e. how they contribute to the goal), which allows a cardinal ranking of the alternatives (Saaty, 2006).

Intensity of Importance	Definition	Explanation
1	Equal Important	Two elements contribute equally to the objective.
3	Weak Importance	Experience and judgment slightly favor one element over another.
5	Strong Importance	Experience and judgment strongly favor one element over another.
7	Very Strong Importance	One element is favored over another; its dominance is demonstrated in practice.
9	Absolute Importance	The evidence favoring one element over another is of the highest possible order of affirmation
Intermediate values used to interpolate between adjacent scale values		

Table1: A nine point scale for pair-wise comparison

2.1. AHP Algorithm

Ideally the judgment matrix should have the following properties:

$$\begin{aligned}
 a_{ii} &= 1 \\
 a_{ij} &= \frac{1}{a_{ji}} \\
 a_{ij} &= a_{ik} a_{kj}
 \end{aligned}$$

According to these properties, the ideal structure of the judgment matrix is:

$$\begin{bmatrix}
 1 & \frac{w_1}{w_2} & \frac{w_1}{w_3} & \frac{w_1}{w_4} & \frac{w_1}{w_5} & \frac{w_1}{w_6} & \dots & \dots & \frac{w_1}{w_n} \\
 \frac{w_2}{w_1} & 1 & \frac{w_2}{w_3} & \frac{w_2}{w_4} & \frac{w_2}{w_5} & \frac{w_2}{w_6} & \dots & \dots & \frac{w_2}{w_n} \\
 \frac{w_3}{w_1} & \frac{w_3}{w_2} & 1 & \frac{w_3}{w_4} & \frac{w_3}{w_5} & \frac{w_3}{w_6} & \dots & \dots & \frac{w_3}{w_n} \\
 \frac{w_4}{w_1} & \frac{w_4}{w_2} & \frac{w_4}{w_3} & 1 & \frac{w_4}{w_5} & \frac{w_4}{w_6} & \dots & \dots & \frac{w_4}{w_n} \\
 \frac{w_5}{w_1} & \frac{w_5}{w_2} & \frac{w_5}{w_3} & \frac{w_5}{w_4} & 1 & \frac{w_5}{w_6} & \dots & \dots & \frac{w_5}{w_n} \\
 \frac{w_6}{w_1} & \frac{w_6}{w_2} & \frac{w_6}{w_3} & \frac{w_6}{w_4} & \frac{w_6}{w_5} & 1 & \dots & \dots & \frac{w_6}{w_n} \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \frac{w_n}{w_4} & \frac{w_n}{w_5} & \frac{w_n}{w_6} & \dots & \dots & 1
 \end{bmatrix}$$

Here the vector $w^T = (w_1, w_2, w_3, w_4, w_5, w_6, \dots, w_n)$ denotes the weights of the alternatives resulting from the judgment. The ideal judgment matrix has the following property:

$$A * w = nw$$

Where n denotes the number of alternatives. This means that under ideal conditions, the vector of the weights is equal to the principal eigenvector.

In practical situations, the judgment matrix does not respect the transitivity condition, i.e., the condition $a_{ij} = a_{ik} a_{kj}$ is usually violated. Saaty suggests then the principal eigenvector of the real matrix should be the weights vectors. In the case of non-ideal judgment matrix, this vector is given by:

$$w = \lim_{k \rightarrow \infty} \left(\frac{A^k e}{e^T A^k e} \right)$$

Where $e^T = (1,1,1,1,...,1)$

An alternative calculation of the weights called the logarithm least squares method (**LLSM**) has been suggested and is given by:

$$w_i = \frac{\left(\prod_{j=1}^{j=n} a_{ij} \right)^{\frac{1}{n}}}{\sum_{i=1}^{i=n} \left(\prod_{j=1}^{j=n} a_{ij} \right)^{\frac{1}{n}}}, i = 1, 2, \dots, n.$$

The same procedure can be applied to rank criteria too.

There is substantial debate regarding the most suitable way to calculate the weights (the eigenvector method - EVM or the logarithm least squares - LLSM method). Many papers have been indicating their advantages and shortcomings, but so far there no clear cut recommendation regarding the best method.

3 AHP User's Guide

In order to visualize the AHP concept into an applicable decision making tool, EWRE developed an AHP application based on Saaty's theory by calculating the weights, both with the eigenvector method and the LLSM method. This application builds the simplest decision model that has a goal, criteria and alternatives, make judgments (paired comparisons), and compute the results to find the best alternative. The goal, the criteria and the alternatives are all elements in the decision problem, or nodes in the model.

The AHP application is MFC Dialog based, that is installed from a cd, and runs on window systems. The Application uses XML to save/load dedicated project files.

The AHP application is divided into four different Tabs - Settings, Criteria Weighting, Judgement and Results. In the first three phases active participation of the decision makers is needed.

1. A preliminary procedure is to construct and represent the elements of a specific problem, which is defined by the decision makers, in a **Settings-Tab**. In this phase, it is needed to specify the overall goal and breakdown it to a set of criteria. The user determines the number of criteria and describes each one of them. Each decision alternative is also described in this phase. Therefore, all the element in the hierarchy are considered to be independent of all the others.

Setting-Tab →

AHP

File Settings Criteria Weighting Judgement Results

Number Of Criteria: 4 → **Set number of Criteria**

Method Of Calculation: Logarithmic Least Squares

☒ AntiSymetric Matrix → **Choose between the EVM or the LLSM**

	Description
C1	Aquifer Protection
C2	Water Supply/Demand Gap
C3	Maximum Economic Profit
C4	Water Supply/Demand Gap on Peak day

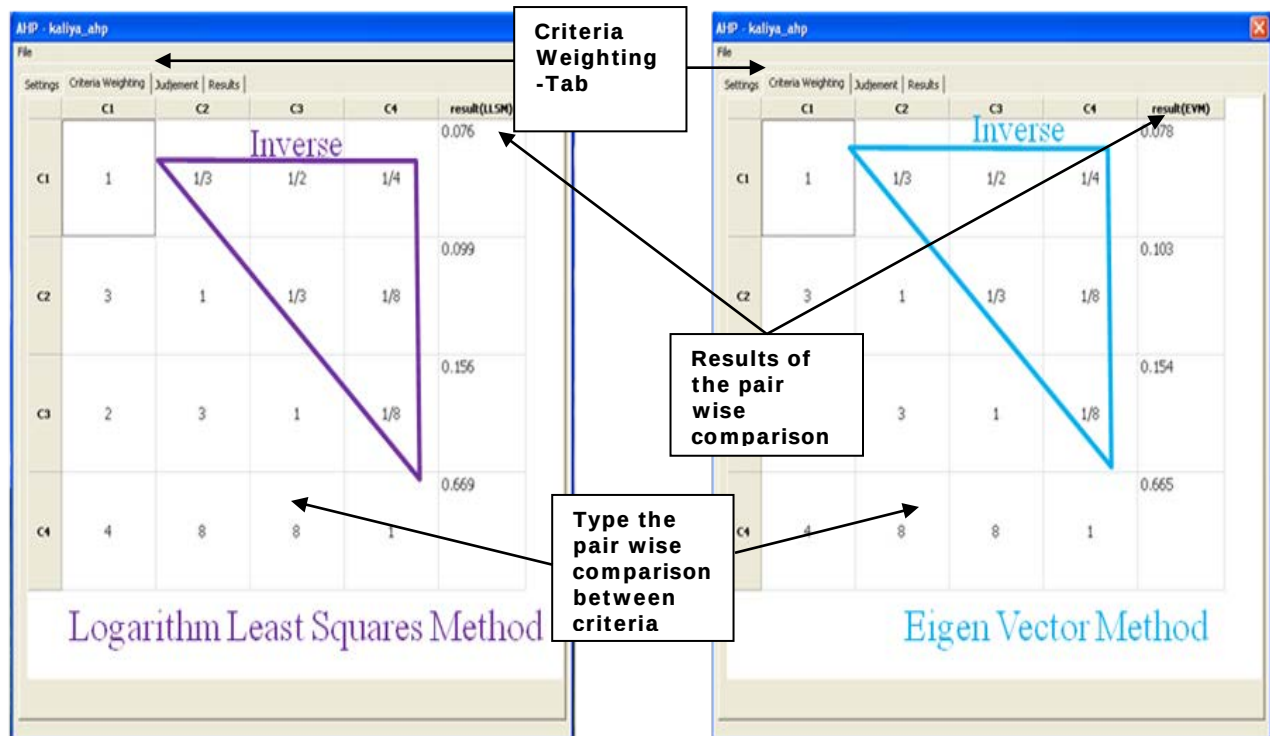
→ **Type description of the criteria**

Number Of Alternatives: 5 → **Set number of Alternatives**

	Description
A1	Only Fresh Water
A2	Fresh Water + Brackish Water
A3	Fresh Water + Effluent
A4	Fresh Water + (Fresh + Brackish)
A5	Fresh Water + (Brackish + Effluent)

→ **Type description of the alternatives**

2. In the **Criteria Weighting-Tab** the decision maker conducts a pair-wise comparisons of the criteria elements in a comparison matrix. This procedure is aimed to evaluate the criteria according to his/her subjective preference. A preferences is always between two criteria on a ratio scale from one to nine. The inverse preference between the two criteria will be automatically shown in the opposite diagonal. The criteria preference results are calculated on the fly.



3. In the **Judgment-Tab**, a pair-wise comparisons of all alternative elements is conducted by the decision makers, for a given criterion (i.e. there is a sub-Tab for each criterion). In other words, a criteria comparison matrix is created respectively to a set of criteria specified by the decision makers in the Settings-Bar). The alternative preference results are calculated on the fly.

Judgment -Tab

	A1	A2	A3	A4	A5	result(EVM)
A1	1	1/5	1/9	1/2	1/7	0.038
A2	5	1	1/4	2	1/5	0.122
A3	9	4	1	3	2	0.413
A4	2	1/2	1/3	1	1/5	0.080
A5	7	5	1/2	5	1	0.347

Type the pair wise comparison between Alternative

Different criteria bar for each comparison matrix

Criteria bar: C1 C2 C3 C4

4. **Results-bar:** The ratings of the decision makers are arranged as numerical numbers as relative weights. Relative weights for all elements of the hierarchy are calculated in this last phase. The element performances are weighted with the relative weights of criteria, and added to an overall priority for each alternative.

AHP - kaliya_ahp

File

Settings | Criteria Weighting | Judgement | Results ← **Results-Tab**

Calculation Method :

	Weight	A1	A2	A3	A4	A5
C1	0.078	0.003	0.010	0.032	0.006	0.027
C2	0.103	0.016	0.009	0.009	0.019	0.049
C3	0.154	0.007	0.016	0.047	0.014	0.071
C4	0.665	0.031	0.033	0.158	0.077	0.366
		0.057	0.068	0.246	0.116	0.513

Results of the criteria pair wise comparison

Final results for all alternative pair wise comparison

4 Electre Tri Algorithm

4.1 Overall view

Electre TRI is an algorithm designed to sort a set of alternatives into a small set of categories. Once the process is completed, the decision maker could choose a single alternative among those assigned to the best category, or continue with an additional process so as to refine the sorting of the best alternatives.

Figure 3 illustrates a simplified geometric model of a realization of a sorting decision problem:

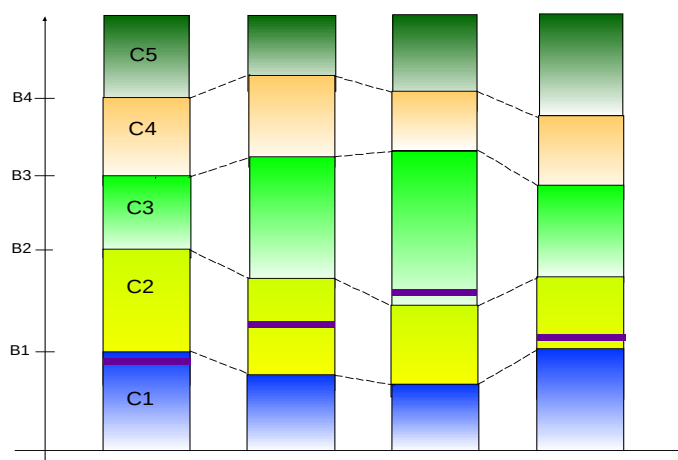


Figure 2: A simplified geometric model of the Electre Tri sorting problem.

To which of the following colored strips does the purple curve fit in the best way for all rectangles?

In Electre TRI the rectangles are the criteria. Each criterion is defined by its range of feasible values (e.g. 0-100), a range which is divided into sub sections (e.g. the blue section C1, whose boundary is B1 in Figure 3). The subsections are the categories. The purple line represents an alternative which has a different value for each of the criteria. In Figure 3 the purple alternative belongs to the blue subsection in criterion 01, to the yellow subsection in criterion 02, etc. The goal is to associate an alternative to a category. Intuitively, it is clear that the purple alternative in Figure 3 belongs to the yellow category.

A criterion defined as well by its weight, i.e. its relative importance compared to other criteria. In Figure 3 the weights are not modeled.

One of the key aspects of Electre TRI is the refinement of the definition of a sharp mathematical boundary between two categories. The motivation of the refinement is the intuitive feeling that even if the purple alternative belongs to the blue category in criterion 01, it is very close to the yellow category. Electre TRI defines “very close” or “close” in a quantitative way. These definitions are comprised into the algorithm which assigns a category to any suggested alternative.

4.2 The Algorithm in detail

4.2.1 Definitions of parameters

The following parameters are defined for each criterion and depicted in Figure 4:

For each criterion, b_j (where j ranges between 1 and N) are the values defining the boundaries of the categories, called profiles, b_0 and b_N are the minimal and maximal feasible values for the criterion. b_j is the upper value of the j th category and the lower value of the $(j+1)$ th category. There are b_N categories altogether.

$1 \leq j \leq (b_N - 1)$ The subsequent definitions and calculations refer to the values of the internal profiles only, where:

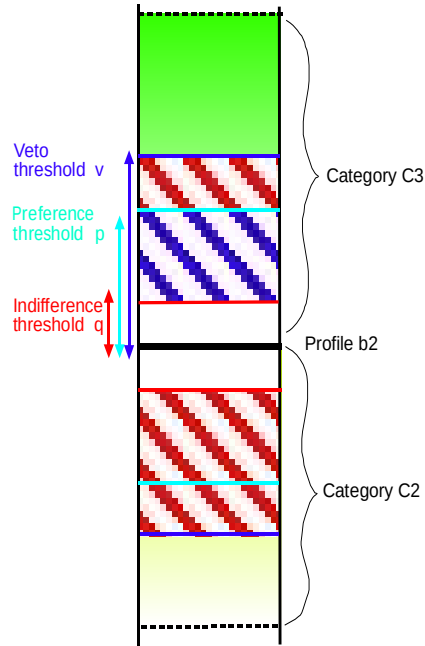


Figure 3: Graphic representation of a criterion and associated parameters.

Each (internal) profile is associated with three additional numbers, q_j is the indifference threshold, p_j is the preference threshold and v_j is the veto threshold. These thresholds enable a continuous transition between the categories, and subsequently a refined, smooth decision regarding the assignment of a number to a category.

4.2.1.1 Directions of preferred values in categories

All the following explanations refer to criteria where the most preferred value is the largest value. In case that the opposite is true (as with prices of items for buyers) a simple normalizing transformation is carried on all the parameters supplied by the user for that criterion so the internal representation is the same as the increasing criteria. Therefore no explicit explanation is carried on for this case.

4.2.1.2 Indifference thresholds

For each (internal) profile b_j , q_j is the indifference threshold, i.e. it defines the width of the strip around the profile where the alternatives will be considered equal or larger to b_j and b_j will be considered equal or larger to an alternative.

4.2.1.3 Preference thresholds

p_j is the preference threshold. An alternative located below between the profile, between the indifference and the preference thresholds will be considered equal or larger than the profile (i.e. fit to be assigned to the category above) with a decreasing consent. When an alternative is located below the preference threshold then the consent becomes zero.

4.2.1.4 Veto thresholds

v_j is the veto threshold. When an alternative is located below the veto threshold then an actual objection to the decision associating the alternative with the category above is recorded. When it is between the veto and the preference thresholds then the objection decreases and when it is above the preference threshold the objection becomes zero. The veto is stronger than mere zero consent in the sense that it puts a veto on the decision to associate the alternative with the category above even if there is strong consent considering all the other criteria.

4.2.2 Concordance indices

Given an alternative value for a specific criterion, denoted by a , the concordance index denoted by $C(a, b_j)$, expresses the extent of the agreement with the statement “ a is at least as good as b_j ”.

It is defined as follows:

$$\begin{aligned} a \leq (b_j - p_j) & \Rightarrow C(a, b_j) = 0 \\ (b_j - p_j) \leq a \leq (b_j - q_j) & \Rightarrow C(a, b_j) \text{ varies linearly between 0 and 1.} \\ a \geq (b_j - q_j) & \Rightarrow C(a, b_j) = 1 \end{aligned}$$

The concordance index denoted $C(b_j, a)$ expresses the extent of the agreement with the statement “ b_j is at least as good as a ”.

It is defined as follows:

$$\begin{aligned} a_j \geq (b_j + p_j) & \Rightarrow C(b_j, a) = 0 \\ (b_j + q_j) \leq a \leq (b_j + p_j) & \Rightarrow C(b_j, a) \text{ varies linearly between 0 and 1.} \\ a \leq (b_j + q_j) & \Rightarrow C(b_j, a) = 1 \end{aligned}$$

4.2.3 Discordance indices

Given an alternative value for a specific criterion, denoted by a , the discordance index (denoted $d(a, b_j)$) expresses the extent of the objection to the statement "a is at least as good as b_j ".

It is defined as follows:

$$a \geq (b_j - p_j) \Rightarrow d(a, b_j) = 0$$

$$(b_j - v_j) \leq a \leq (b_j - p_j) \Rightarrow d(a, b_j) \text{ varies linearly between 0 and 1.}$$

$$a \leq (b_j - v_j) \Rightarrow d(a, b_j) = 1$$

The discordance index (denoted $d(b_j, a)$) expresses the extent of the objection to the statement " b_j is at least as good as a ".

It is defined as follows:

$$a \leq (b_j + p_j) \Rightarrow d(b_j, a) = 0$$

$$(b_j + p_j) \leq a \leq (b_j + v_j) \Rightarrow d(b_j, a) \text{ varies linearly between 0 and 1.}$$

$$a \geq (b_j + v_j) \Rightarrow d(b_j, a) = 1$$

Given an alternative, the decision process for each profile in every criterion separately ends with four numbers quantifying the consent to the statements "the alternative is at least as good as the profile", "The profile is at least as good as the alternative" and the objection to the same statements.

4.2.4 The decision process for multiple criteria

4.2.4.1 Parameters

The following parameters are associated with the final decision:

There are **M** criteria.

Weights: Each criterion has a weight (denoted $k_i, 1 \leq i \leq M$) associated with it, which quantifies its relative importance.

Decision cut: A number associated with the process as a whole, (denoted λ cut) is needed to make a crisp final decision regarding the comparison between an alternative and a profile.

Mode of decision: Two modes denoted "pessimistic" and "optimistic", can be chosen for the final decision regarding the assignment of an alternative to a specific category.

4.2.4.2 Final decision steps

Global concordance indices

The Integration of the concordance indices for all criteria is carried on through the calculation of two global concordance indices. The assertion "a is at least as good as bj for all criteria" (and every profile separately) is calculated via

$$C_G(a, bj) = \frac{\sum_{i=1}^M k_i c_i(a, bj)}{\sum_{i=1}^M k_i} \quad \text{Where the index } i \text{ has been added to the notation } C(a, bj) \text{ to}$$

indicate the i-th criterion.

Similarly the assertion "bj is at least as good as a for all criteria"

$$C_G(bj, a) = \frac{\sum_{i=1}^M k_i c_i(bj, a)}{\sum_{i=1}^M k_i}$$

The degree of credibility indices

The objection indices are compiled within the calculation through the definition of the credibility indices σ . They are defined as follows:

$$\sigma(a, bj) = C_G(a, bj),$$

when no criteria are discordant, i.e. all $d(a, bj)$ are zero.

$$\sigma(a, bj) = 0,$$

When at least one discordant index is equal to 1, i.e. opposes a veto to the assertion.

$$\sigma(a, bj) = C_G(a, bj) \prod_{i=1}^M \frac{1 - d_i(a, bj)}{1 - C_G(a, bj)}$$

where the terms in the multiplication are calculated only if $d_i(a, bj) > C_G(a, bj)$.

The other credibility index is calculated equivalently.

4.2.4.3 The relation between an alternative and a profile

Presently, there are two resulting parameters – the credibility indices of the assertions "the alternative is at least as good as the profile", and "the profile is at least as good as the alternative", which are associated with every profile for a given alternative. The translation of these parameters into an assignment to a category using the decision cut λ is explained below:

When a credibility index satisfies a relation of the form

$$\sigma(a, b_j) \geq \lambda ,$$

Then the assertion "the alternative a is at least as good as the profile b_j " is finally accepted. This assertion is denoted $(a \succeq b_j)$. The possible relations between an alternative and a profile and their notations are:

- Indifference denoted I .
- Preference, denoted $\succ$.
- Incomparability, denoted R .

Therefore

- $a \succeq b_j$ and $b_j \succeq a \iff a I b_j$
- $a \succeq b_j$ and not $b_j \succeq a \iff a \succ b_j$
- $b_j \succeq a$ and not $a \succeq b_j \iff b_j \succ a$
- not $a \succeq b_j$ and not $b_j \succeq a \iff a R b_j$

4.2.4.4 The final assignment procedure

There are two modes for the assignment decision:

- **Pessimistic mode:** Compare the relations between an alternative and each of the profiles starting from the highest downwards. The alternative is assigned to the category above the first profile such that $a \succ b_j$, i.e. to the $(j+1)$ category.
- **Optimistic mode:** Compare the relations between an alternative and each of the profiles starting from the lowest upwards. The alternative is assigned to the category below the first profile such that $b_j \succ a$, i.e. to the j th category.

5 Electre Tri Assistant Algorithm

5.1 Overall view

Using Electre Tri means supplying many parameters: the range of the criteria, their weights, a set of profiles for each criterion plus three additional threshold parameters for each profile in each criterion and finally the value of the decision cut. Often a potential MCDM user does not have a definite idea regarding the exact values of all these parameters. However, she may possess an opinion regarding the assignment of a simple “learning” set of examples to categories. This means that the user can supply a set of these learning examples such that each has a value for each criterion, and an association with a category. These examples may be realistic, or even partially virtual. The idea behind Electre Tri assistant is using a mathematical computed algorithm to infer the parameters of the basic Electre Tri algorithm using the information of the assigned examples. This sophisticated process involves optimization of the values of the parameters with respect to constraints.

The type of problem to be solved belongs to constrained non-linear optimization problems, i.e.

$$\begin{aligned} &\text{Min. } f(x) \\ &\text{s.t.} \\ &g_j(x) = 0, \quad j=1, \dots, m_e \\ &g_j(x) \geq 0, \quad j=m_e+1, \dots, m \\ &x_l \leq x \leq x_u \quad x \in R^n \end{aligned}$$

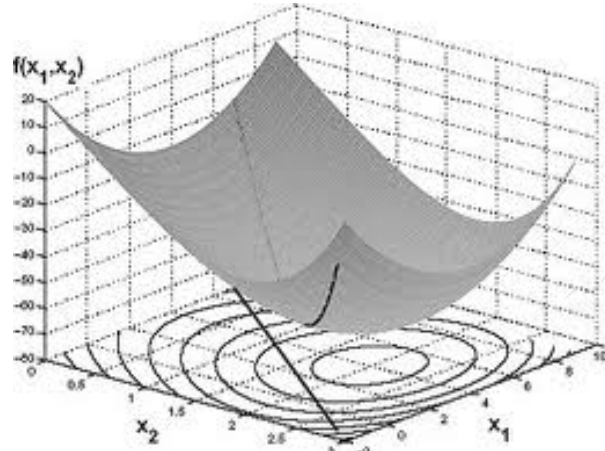


Figure 4: Illustration of an optimization problem

In Figure 5 such a problem where the function $f(x)$ has only two independent variables (i.e. $n=2$) and the single equality constraint is a straight line is depicted. The solution, which is the minimal point on the surface when one traces the straight line along the plane, is located around the point $(2, 1.75)$.

The numerical solution method and implementation software NLPQLP were developed by K. Schittkowski.

5.2 The Algorithm in detail

The optimization problem for the inference of Electre Tri parameters from examples is written below:

min.

$$-(\alpha + \varepsilon \sum (x_i + y_i))$$

s.t.

$$x_i \geq \alpha,$$

$$y_i \geq \alpha,$$

$$\sigma_\pi(a_i, b_{hi-1}) - x_i = \lambda,$$

$$\sigma_\pi(a_i, b_{hi}) - y_i = \lambda,$$

$$\lambda \in [0.5 \quad 1],$$

$$g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1}),$$

$$p_j(b_h) \geq q_j(b_h).$$

$$k_j \geq 0, \quad q_j \geq 0$$

Where the unknowns are the following parameters:

$g_j(b_k)$ - the value of the k^{th} profile for the j^{th} criterion.

$p_j(b_k)$ - preference values,

$q_j(b_k)$ - indifference values,

k_j - weights,

x_i and y_i - distances of the global consents from the decision cut.

In order to keep the size of the problem small the veto values are calculated at the beginning of the algorithm and are not part of the optimization process.

a_i - The known data of the values of the alternatives.

The crucial point is that the goal is finding parameters which maximize the distances of the global consents for each example/alternative from the decision cut. The problem is phrased as a minimization problem of the negative function in order to conform to Schittkowski implementation within the software. The constants ε and α increase the robustness of the optimization process. The constraints are straightforward: the weights have to be non-negative, as well as the indifference thresholds, the preferences should be larger than the indifferences, and a profile should be larger than the sum of the smaller profile and the preference values of both, in order to prevent a contradiction.

6 ELECTRE TRI Assistant User's Guide

In order to transform the Electre Tri assistant concept into an applicable decision making tool, EWRE developed an application, implementing the optimization process.

Similar to the AHP application, the Electre application is MFC Dialog based, that is installed from a cd, and runs on window systems. The Application uses XML to save/load dedicated project files.

The Electre Tri application is divided into four different phases– definitions of the categories & examples by the user, calculation of the parameters, definition of the alternatives, and calculation of the required Result, i.e. the sorting of the alternatives into the categories. In the first and the third phases active participation of the decision makers is needed.

The screenshot shows the 'Electre_TRI_2' application window. It features a menu bar with 'File' and a toolbar with standard window controls. The main area contains a table with columns C1, C2, C3, and C4. The rows are organized into four phases:

- Phase 2: results of „Assist“** (indicated by a bracket on the right): Includes rows for Description, Weight, Best, Profile3, Profile2, Profile1, and Worst.
- Phase 4 Results of „Evaluate“** (indicated by a bracket on the right): Includes rows for Alternatives, Alternative1, Alternative2, Examples, and Category.
- Phase 1** (indicated by a bracket on the right): Includes rows for Best, Example1, Example2, Example3, and Worst.
- Phase 3** (indicated by a bracket on the left): Points to the 'Alternatives' row.

At the bottom of the window, there are buttons for 'Add', 'Remove', 'Assist', and 'Evaluate', along with a 'Decision cut' field set to 0.75.

	C1	C2	C3	C4		
Description						
Weight						
Best						
Profile3						
Profile2						
Profile1						
Worst						
Alternatives					Result(opt)	Result(pess)
Alternative1						
Alternative2						
Examples					Category	
Best						
Example1						
Example2						
Example3						
Worst						

Phase 1

1. Fill names of the criteria. If an additional criterion is needed, press "Add", and choose criterion, if a criterion should be deleted, click on it, press "Remove" and choose criterion.
2. Fill bounds of each criterion.
3. For each basic example, fill a value for each criterion, and assign to a category. The values in each column/ criterion should be between the bounds for that criterion.
4. The last step in this phase is to click on the "Assist" button in order to calculate profiles for each criterion.

Electre_III_2

File

	C1	C2	C3	C4	C5		
Description	Aq. Sus	Certainty	Cost	Env. Sus	Peak Dis		
Weight							
Best	100.00	100.00	100.00	100.00	100.00		
Profile3							
Profile2							
Profile1							
Worst							
Alternatives						Result(opt)	Result(pess)
Alternative1							
Alternative2							
Examples						Category	
Best	100.00	100.00	100.00	100.00	100.00		
Example1	60.00	10.00	20.00	80.00	60.00	1	
Example2	80.00	70.00	40.00	10.00	65	2	
Example3	65.00	40.00	30.00	80.00	65.00	1	
Example4	90	80	50	50	70	2	
Example5	90	80	70	60	90	3	
Example6	100	90	75	50	85	3	
Worst	0	0	0	0	0		

1. Type names for each criteria

2. Type Bounds: Worst and Best

3. Type values for every example & for each criterion, and assignment of each example to a category

4. Click to calculate parameters of profiles

Assist

Add Remove Evaluate

Decision cut 0.75

Click to Add or Remove a criterion, an example, or an alternative

Phase 2

The result of pressing „Assist“ is the calculation of all parameters of the Electre Tri. The criteria are weighted and the parameters of the profiles are set for every criterion. The bounds - worst and best, which were given to each criterion in the first phase, are shown again.

In this phase, the user is passive.

The screenshot shows the 'Electre_III_2' application window. It contains a table with the following data:

	C1	C2	C3	C4	C5		
Description	Aq. Sus	Certainty	Cost	Env. Sus	Peak Dis		
Weight	1.24	1.24	1.24	0.10	1.23		
Best	100.00	100.00	100.00	100.00	100.00		
Profile2	90.03 q=4.50 p=8.97 v=9.97	80.00 q=4.00 p=8.00 v=20.00	58.75 q=2.94 p=5.88 v=23.75	49.22 q=0.00 p=0.00 v=0.94	77.94 q=3.72 p=7.31 v=13.38		
Profile1	73.72 q=3.69 p=7.34 v=16.31	50.00 q=2.50 p=5.00 v=30.00	35.00 q=1.75 p=3.50 v=23.75	48.28 q=0.80 p=0.90 v=0.94	64.56 q=3.08 p=6.06 v=13.38		
Worst	0.00	0.00	0.00	0.00	0.00		
Alternatives						Result(opt)	Result(pess)
Alternative1							
Alternative2							
Examples						Category	
Best	100.00	100.00	100.00	100.00	100.00		
Example1	60.00	10.00	20.00	80.00	60.00	1	
Example2	80.00	70.00	40.00	10.00	65.00	2	
Example3	65.00	40.00	30.00	80.00	65.00	1	
Example4	90.00	80.00	50.00	50.00	70.00	2	
Example5	90.00	80.00	70.00	60.00	90.00	3	
Example6	100.00	90.00	75.00	50.00	85.00	3	
Worst	0.00	0.00	0.00	0.00	0.00		

At the bottom of the window, there is a control panel with the following elements:

- An 'Assist' button.
- 'Add' and 'Remove' buttons.
- An 'Evaluate' button.
- A 'Decision cut' field with a value of 0.75.

Two callouts are present:

- 'Each criteria is weighed' pointing to the 'Weight' row.
- 'The parameters of the profiles are calculated' pointing to the 'Profile1' and 'Profile2' rows.

Phase 3

1. Fill values for each alternative, for each criterion. If an additional alternative is needed, click "Add" and choose alternative. To remove an alternative, mark it, click "Remove" button and choose "alternative".
2. Click "Evaluate".

Electre_III_2

File

	C1	C2	C3	C4	C5		
Description	Aq. Sus	Certainty	Cost	Env. Sus	Peak Dis		
Weight	1.24	1.24	1.24	0.10	1.23		
Best	100.00	100.00	100.00	100.00	100.00		
Profile2	90.03 q=4.50 p=8.97 v=9.97	80.00 q=4.00 p=8.00 v=20.00	58.75 q=2.94 p=5.88 v=23.75	49.22 q=0.00 p=0.00 v=0.94	77.94 q=3.72 p=7.31 v=13.38		
Profile1	73.72 q=3.69 p=7.34 v=16.31	50.00 q=2.50 p=5.00 v=30.00	35.00 q=1.75 p=3.50 v=23.75	48.28 q=0.80 p=0.90 v=0.94	64.56 q=3.08 p=6.06 v=13.38		
Worst	0.00	0.00	0.00	0.00	0.00		
Alternatives						Result(opt)	Result(pess)
Alternative1	55	55	34	49	70		
Alternative2	88	80	62	50	74		
Examples						Category	
Best	100.00	100.00	100.00	100.00	100.00		
Example1	60.00	10.00	20.00	80.00	60.00	1	
Example2	80.00	70.00	40.00	10.00	65.00	2	
Example3	65.00	40.00	30.00	80.00	65.00	1	
Example4	90.00	80.00	50.00	50.00	70.00	2	
Example5	90.00	80.00	70.00	60.00	90.00	3	
Example6	100.00	90.00	75.00	50.00	85.00	3	
Worst	0.00	0.00	0.00	0.00	0.00		

Assist

Add Remove Evaluate

Decision cut 0.75

Click to Add or Remove a criterion, an example, or an alternative

Type values for each alternative

Phase 4

Final calculation is completed in this last phase. The result of pressing “Evaluate” button are shown under “Result (opt)”, which is the short form of “optimistic strategy” and “Result (pess)” for pessimistic strategy.

In this specific example, both strategies assigned the 2nd alternative to category 3, the best category, and it is easy to see that most values of that alternative are high. Whereas, the 1st alternative is assigned to category 2 by the optimistic strategy, and to category 1 by the pessimistic strategy. It is easy to see that 2 values of this alternative are close to profile 1, which divides category 1 and 2, one is well below and two are slightly above, so intuitively it is not a clear case.

The advantage of seeing the results of two strategies is pronounced in the cases where they differ by a large number of categories. This may happen when an alternative has many low values and many high values as well, so in fact it does not belong to any category, and the discrepancy between the results of the two strategies is the test for this event.

Electre_III_2

	C1	C2	C3	C4	C5		
Description	Aq. Sus	Certainty	Cost	Env. Sus	Peak Dis		
Weight	1.24	1.24	1.24	0.10	1.23		
Best	100.00	100.00	100.00	100.00	100.00		
Profile2	90.03 q=4.50 p=8.97 v=9.97	80.00 q=4.00 p=8.00 v=20.00	58.75 q=2.94 p=5.88 v=23.75	49.22 q=0.00 p=0.00 v=0.94	77.94 q=3.72 p=7.31 v=13.38		
Profile1	73.72 q=3.69 p=7.34 v=16.31	50.00 q=2.50 p=5.00 v=30.00	35.00 q=1.75 p=3.50 v=23.75	48.28 q=0.80 p=0.90 v=0.94	64.56 q=3.08 p=6.06 v=13.38		
Worst	0.00	0.00	0.00	0.00	0.00		
Alternatives						Result(opt)	Result(pess)
Alternative1	55	55	34	49	70	2	1
Alternative2	88	80	62	50	74	3	3
Examples						Category	
Best	100.00	100.00	100.00	100.00	100.00		
Example1	60.00	10.00	20.00	80.00	60.00	1	
Example2	80.00	70.00	40.00	10.00	65.00	2	
Example3	65.00	40.00	30.00	80.00	65.00	1	
Example4	90.00	80.00	50.00	50.00	70.00	2	
Example5	90.00	80.00	70.00	60.00	90.00	3	
Example6	100.00	90.00	75.00	50.00	85.00	3	
Worst	0.00	0.00	0.00	0.00	0.00		

Assist

Add Remove Evaluate Decision cut 0.75

Click to calculate final results

Final results for all alternatives